Proceedings of the International Conference on Aerospace Science and Technology
26 - 28 June 2008, Bangalore, India

INCAST 2008-058

FINITE ELEMENT STUDIES ON SUPERSONIC PANEL FLUTTER UNDER HIGH THERMAL ENVIRONMENT WITH ARBITRARY FLOW DIRECTION

Somenath Mukherjee, M Manjuprasad, S Deepa Sakravarthini, R Avinash
Structural Technologies Division, National Aerospace Laboratories, Bangalore 560017, India,
somu@css.nal.res.in, manjuprasad@css.nal.res.in, varthini.sar@gmail.com, avinash_r9@yahoo.com

ABSTRACT: Panels of re-entry vehicles are subjected to a wide range of flow conditions during ascent and re-entry phases. The flow can vary from subsonic continuum flow to hypersonic rarefied flow with wide ranging dynamic pressure and associated aerodynamic heating. One of the main design considerations is the assurance of safety against panel flutter under the flow conditions characterized by harsh thermal environment. This paper presents the work carried out at NAL to estimate the effects of a thermal profile in lowering the critical dynamic pressure (flutter boundary) of flat rectangular panels subjected to supersonic flow. A finite element formulation (employing the Kirchoff plate C^1 bending element) has been developed here for supersonic flutter analysis of simply supported rectangular panels without in-plane edge constraints subjected to an assumed parabolic thermal profile that can result from any residual heat seeping into the metallic panels through the thermal protection systems. The piston theory is used for aerodynamic pressure computations, and provision is made to take into account the effect of arbitrary flow directions with respect to the panel edges. The results generated using the in-house finite element code and also the MSC NASTRAN software are in good agreement with analytical results. From the analysis of the results for various flow directions it has been observed that the flow along the longer side of any panel is most critical. It has been shown that for simply supported panels with no in-plane edge constraints, the thermal gradients (from the assumed parabolic profiles) can cause a drastic fall in the flutter boundary due to in-plane thermal stresses that effectively reduce structural stiffness. The present study will be useful for the purpose of panel design in re-entry launch vehicles and supersonic fighter aircrafts.

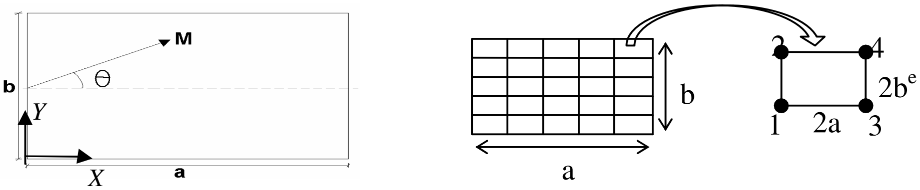
1. INTRODUCTION

Despite a thermal protection system layer (TPS), some temperature rise during supersonic/hypersonic flight is anticipated in the metallic panels that form the skins of re-entry launch vehicles. This can engender in-plane compressive stress resultants. This relatively small temperature rise can have deleterious impact upon the flutter boundary and thus must be taken into account for designing panels subjected to harsh supersonic flow conditions. Ashley and Zartarian [1] have presented a method to estimate the aerodynamic forces acting on panels subjected to supersonic airflow. Early experimental and theoretical studies of the flutter behavior of buckled plates have been presented in references [2-5]. The results and observation of the investigation on the deleterious thermal effects on the flutter boundary of panels in supersonic flow along the panel edges have been reported [6]. The finite element method has been earlier applied [7] for supersonic panel flutter analysis without thermal effects, using a conforming quadrilateral (CQ) element for *arbitrary* flow directions.

The objective of the present work is to investigate the effects of an assumed parabolic thermal profile on the flutter boundary of isotropic simply supported rectangular panels using a finite element (FE) formulation developed at NAL and also by using an FE model of the problem through the NASTRAN software. Results generated using this FE formulation are in good agreement with those obtained from NASTRAN. The aerodynamic forces from the supersonic airflow have been modeled using the piston theory aerodynamics of reference [1] for theoretical analysis. The theoretical concepts of thermal effects presented earlier [6] have been extended here for developing a FE formulation for the problem with arbitrary flow directions. In the present work, the four noded C^1 Kirchoff's plate element [8] has been used. The detailed analysis for supersonic panel flutter under thermal environment with flow in arbitrary direction has been presented by Mukherjee et al [9,10].

2. FORMULATION FOR SUPERSONIC PANEL FLUTTER

The panel configuration ($a\times b$, thickness h) of the simply supported panel and its finite element discretization are shown in Fig 1. It is subjected to a supersonic airflow at Mach number 'M' along the direction making an angle 'θ' with the edge 'a' of the panel.

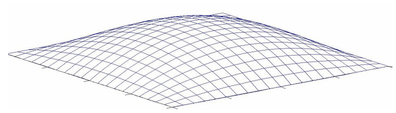


(i) **(ii)**

Figure 1. (*i*) Panel under flow along the direction of angle 'θ' with the edge 'a' and (*ii*) The FEM discretization of the rectangular panel into rectangular elements each of size $2a^e \times 2b^e$.

The panel is subjected to a parabolic temperature distribution in the middle plane as in Fig 2, with temperature difference of ΔT_1 between the center and the edges. The parabolic temperature distribution over the panel is given mathematically by the function

$$T(x,y) = 16\Delta T_1\left(\frac{x}{a}\right)\left(1-\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{y}{b}\right) \tag{1}$$



$$\Delta T_1 = T_{center} - T_{edges}\,.$$

Figure 2. Parabolic Temperature Distribution over the Panel.

2.1. Basic Equations

The equation of motion of the panel under a loading per unit area is given as

$$D\nabla^4 w + N_{xT}\frac{\partial^2 w}{\partial x^2} + 2N_{xyT}\frac{\partial^2 w}{\partial x\partial y} + N_{yT}\frac{\partial^2 w}{\partial y^2} + \rho_{mat} h\frac{\partial^2 w}{\partial t^2} = p \tag{2}$$

Here $w(x,y,t)$ is the dynamic transverse displacement , $D = Eh^3/(12(1-\mu^2))$ is the flexural rigidity of the panel, E is the Young's modulus, μ is the Poisson's ratio and h is the thickness of the plate. The *in-plane* axial stress resultants (N_{xT} and N_{yT} , assumed positive for compression) per unit width along *x*- and *y*-directions of the panel are those resulting from the parabolic temperature distribution while the corresponding shear loading is denoted by N_{xyT}. The mass per unit area of the panel is $\rho_{mat}h$ where ρ_{mat} is the mass density of the panel material. The unsteady aerodynamic pressure load p is obtained by use of linearized, quasi-steady, two-dimensional aerodynamics (Piston theory), originally proposed by Ashley [1],

$$p = -\frac{2q}{\sqrt{M^2-1}}\left(\frac{\partial w}{\partial x}\cos\theta + \frac{\partial w}{\partial y}\sin\theta\right) \tag{3}$$

Here q is the dynamic pressure ($q=\rho_{air}V^2/2$), V is the flow speed and M is the supersonic Mach number.

The thermal stress function $\phi=\phi(x,y)$, given by [6] satisfies the equation $\nabla^4\phi = \alpha Eh\nabla^2 T$ and the condition that the panel be free from thermally induced in-plane normal and shear stresses on the boundaries.

$$\phi(x,y) = C\alpha\, E\, h\, a^2\Delta T_1\left(\frac{x}{a}\right)^2\left(\frac{x}{a}-1\right)^2\left(\frac{y}{b}\right)^2\left(\frac{y}{b}-1\right)^2 \qquad C = -\frac{6\left(1+\beta^2\right)}{1+\left(\frac{4}{7}\right)\beta^2+\beta^4} \quad ,\ \beta = \frac{a}{b} \tag{4}$$

Thus the in-plane stress resultants (see Fig 3) for the parabolic thermal profile are given as [6]

$$N_{xT} = \frac{\partial^2\phi}{\partial y^2} = C\alpha Eha^2\Delta T_1\left(\frac{x}{a}\right)^2\left(\frac{x}{a}-1\right)^2\left(\frac{12y^2}{b^4}-\frac{12y}{b^3}+\frac{2}{b^2}\right)$$

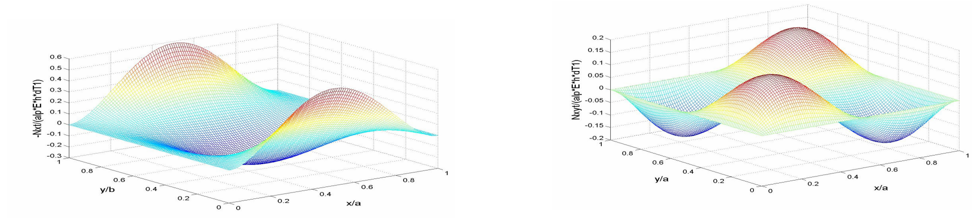
$$N_{yT} = \frac{\partial^2\phi}{\partial x^2} = C\alpha Eha^2\Delta T_1\left(\frac{12x^2}{a^4}-\frac{12x}{a^3}+\frac{2}{a^2}\right)\left(\frac{y}{b}\right)^2\left(\frac{y}{b}-1\right)^2$$

$$N_{xyT} = -\frac{\partial^2\phi}{\partial x\partial y} = -C\alpha Eha^2\Delta T_1\left(\frac{4x^3}{a^4}-\frac{6x^2}{a^3}+\frac{2x}{a^2}\right)\left(\frac{4y^3}{b^4}-\frac{6y^2}{b^3}+\frac{2y}{b^2}\right)$$

where α is the coefficient of thermal expansion. Analytical solutions, using the Galerkin method, have been obtained [6, 9, 10] using

$$w(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{j\omega t} \tag{5}$$

where ω is the circular frequency and t is the time.



(a) **(b)**

Figure 3. Variation of (a) Normal stress resultant N_{xT} , (b) Shear stress resultant N_{xT} for a square panel subjected to parabolic temperature distribution.

2.2 Finite Element Formulation for Supersonic Panel Flutter

For the present work a Quad4 C^1 continuity plate element is selected (see Fig 1). At each node in an element there are three displacement components, *viz.* the transverse displacement w and the slopes about x and y-axes. The transverse displacement in the plate element is thus approximated as

$$w=\left[N^b\right]\left\{d^e\right\}=\left[N_1^b \quad N_2^b \quad N_3^b \quad N_4^b\right]\{d\} \qquad \{d_i\}=\left\{w_i \quad -\left(\frac{\partial w}{\partial y}\right)_i \quad \left(\frac{\partial w}{\partial x}\right)_i\right\}^T \tag{6}$$

where the $[N_i^b]$ are the C^1 shape function matrices (i=1,2,3,4), given in reference [8].

$$\left[N_i^b\right]=\frac{1}{8}\begin{bmatrix}(\xi_o+1)(\eta_o+1)\left(2+\xi_o+\eta_o-\xi^2-\eta^2\right)\\ a^e\xi_i(\xi_o+1)^2(\xi_o-1)(\eta_o+1)\\ b^e\eta_i(\xi_o+1)(\eta_o+1)^2(\eta_o-1)\end{bmatrix}^T \qquad \xi_o=\xi\xi_i\,,\eta_o=\eta\eta_i \qquad \xi=\frac{(x-x_c)}{a^e} \quad , \quad \eta=\frac{(y-y_c)}{b^e} \tag{7}$$

Here (x_c, y_c) are the global co-ordinates of the center of the element. Using Galerkin's method upon the equation of motion (2), one gets first the equation of motion of a typical single element. These equations from all the elements of the domain are then assembled to form the following global (with superscript g) equation

$$\left(\left[K^g\right]+\left[K_s^g\right]+\left[Q^g\right]\right)\left\{d^g\right\}+\left[M^g\right]\left\{\ddot{d}^g\right\}=0 \tag{8}$$

For a typical individual element, the stiffness matrix, the thermally induced matrix, the aerodynamic matrix and the mass matrix respectively are given by

$$\left[K^e\right]=a^e\times b^e\int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]\,d\xi, \qquad \left[K_s^e\right]=a^eb^e\int_{-1}^{1}\int_{-1}^{1}\left(\begin{bmatrix}\frac{1}{a^e}\frac{\partial N^b}{\partial\xi}\\ \frac{1}{b^e}\frac{\partial N^b}{\partial\eta}\end{bmatrix}^T\left[N_m\right]\begin{bmatrix}\frac{1}{a^e}\frac{\partial N^b}{\partial\xi}\\ \frac{1}{b^e}\frac{\partial N^b}{\partial\eta}\end{bmatrix}d\xi\,d\eta\right)$$

$$\left[Q^e\right]=\left[Q_x^e\right]\cos\theta+\left[Q_y^e\right]\sin\theta, \qquad \left[M^e\right]=h\rho_{mat}\int\int\left[N^b\right]^T\left[N^b\right]a^eb^e\,d\xi\,d\eta \tag{9}$$

where

$$\left[Q_x^e\right]=\int_{-1}^{1}\int_{-1}^{1}\frac{2q}{\sqrt{M^2-1}}\left(\left[\frac{1}{a^e}\frac{\partial N^b}{\partial\xi}\right]^T\left[N^b\right]\right)a^e\,b^e\,d\xi\,d\eta, \quad \left[Q_y^e\right]=\int_{-1}^{1}\int_{-1}^{1}\frac{2q}{\sqrt{M^2-1}}\left(\left[\frac{1}{b^e}\frac{\partial N^b}{\partial\eta}\right]^T\left[N^b\right]\right)a^e\,b^e\,d\xi\,d\eta$$

A normal mode superposition method (using the modal matrix $[\phi]$) will now be used to solve equation (8).

$$\{d^g\}=[\phi]\{\nu\}e^{\gamma t}\ , \ \{\ddot{d}^g\}=\gamma^2[\phi]\{\nu\}e^{\gamma t} \qquad [\phi]=[\{\phi_1\},\{\phi_2\},\{\phi_3\},....\{\phi_n\}] \tag{10}$$

Using (10), equation (8) can be brought into the modal domain. For non-trivial solutions, we now have the following eigenvalue problem with $-\gamma^2$ as the eigenvalue,

$$\left|\,\left[K_{Tgen}\right]-\left(-\gamma^2\right)\left[M^g{}_{gen}\right]\right|=0 \tag{11}$$

where $\left[K_{Tgen}\right]=\left[K^g{}_{gen}\right]+\left[K^g{}_{sgen}\right]+\left[Q^g{}_{gen}\right]$. In general, the eigenvalue can be expressed as a complex number, $\gamma=\gamma_r+j\gamma_i$, ($j$=√-1) where the real part γ_r represents the amplitude increase (γ_r>0) or amplitude decrease (γ_r<0) with time, and the imaginary part γ_i is the circular frequency ω. The lowest value of dynamic pressure for γ_r is positive (γ_r>0) for any mode is the critical dynamic pressure q_{cr}.

3. NUMERICAL RESULTS

A numerical study is done by analytical, in-house finite element code and finite element package NASTRAN to determine the supersonic flutter boundary of simply supported aluminium panels of aspect ratios 1, 2, and 7.2. First, the flutter speeds (from modal coalescence of panels modes), for flow along edge *a*, (θ=0) are determined for these specimens *without thermal effects*. These are presented in Table 1, and the $V\omega$ and Vg curves for specimen A are shown in Fig 4. Good agreement of results can be observed.

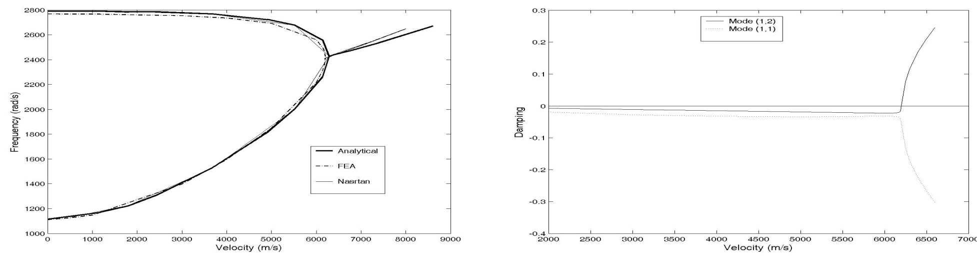


Figure 4. Velocity V (*m/s*) Vs Frequency ω (*rad/s*) and Velocity Vs Damping (*$g=2\gamma_r / \omega$, from NASTRAN*) for a panel of aspect ratio (*a/b*) =1 (specimen A) and air density ρ = 1.225 *kg/m*^{3}.

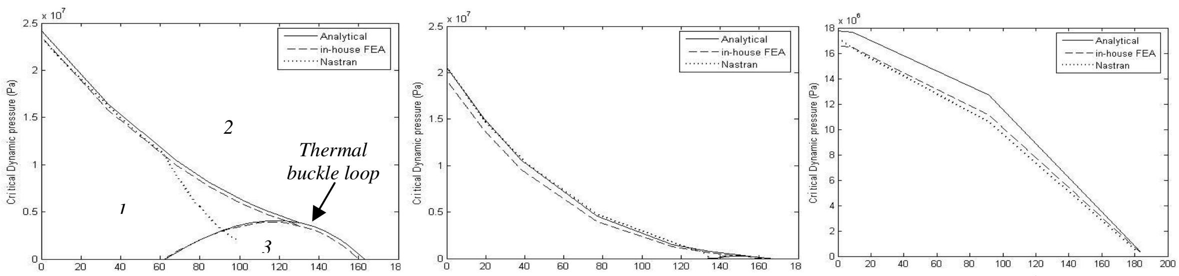
Table 1. Comparison of analytical and finite element method results with NASTRAN for panels of aspect ratios 1, 2 and 7.2 (specimens A, B and C respectively) with flow along edge '*a*', (θ=0).

Specimen	Dimensions (*m*) and aspect ratio (*a/b*)	Non dimensional Critical Dynamic Pressure λ_{cr} $\lambda_{cr}=2\, q_{cr}\, a^3/D$	Air density (kg/m^3)	Critical Dynamic Pressure q_{cr} (*MPa*)	Critical Flow Speed V_{cr} (*m/s*)	Critical Mach number M_{cr}
A	*a* = 0.25 *b* = 0.25 *h* = 0.00232 (*a/b*)=1	512	1.225	*24.2 **23.6 #23.43	*6287.145 **6202.08 #6185.00	*18.492 **18.24 #18.19
B	*a* = 1.0 *b* = 0.50 *h* = 0.007 (*a/b*)=2	1099	1.225	*20.6 **19.2 #20.6	*5796.41 **5593 #5797.50	*17.048 **16.45 #17.05
C	*a* = 0.36 *b* = 0.05 *h* = 0.0011;(*a/b*)=7.2	9387.5	0.715	*17.8 **16.6 #17.2	*7053.518 **6813.055 #6936.589	*20.74 **20.03 #20.40

*Results obtained by analytical formulation; **Results obtained by in-house finite element method formulation; #Results obtained by NASTRAN. For aluminium, Young's Elasticity, E = 70x10^9 *N/m*^{2}, Poisson's ratio, μ = 0.3, Coefficient of thermal expansion $\alpha = 2.3\text{x}10^{-5}\,/^oC$ and material density ρ_{mat} = 2764 *kg/m*^{3}.

3.1 Thermal Effects on Flutter Boundary

The lowering of critical dynamic pressure for flow on panel (specimens A,B,C) along edge *a* with increase in the parabolic temperature profile distribution ΔT_1(^{0}C) is shown in Fig 5. Nastran results are in good agreement with both analytical and in-house FEA results till the thermal buckling point, beyond which the NASTRAN results show more drastic fall of critical conditions than the predicted by other methods. This can be attributed to the non-linear formulation of NASTRAN, while the in-house FEA and analytical formulations are linear.



(a) (b) (c)

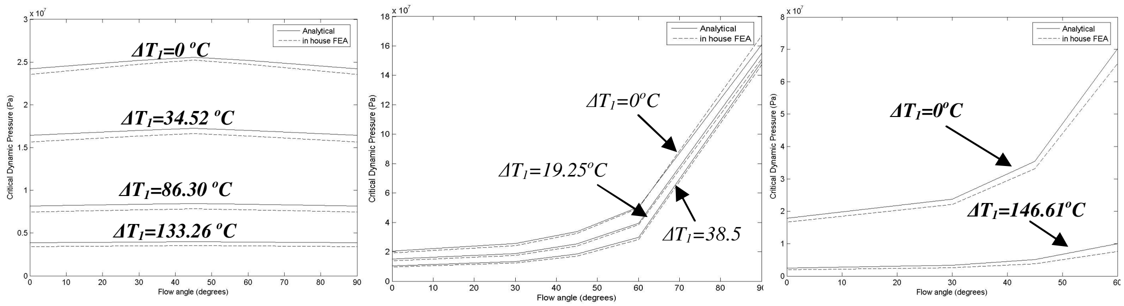
Figure 5. Variation of critical dynamic pressure q_{cr} (in N/m^2), with flow along edge '*a*' for various thermal profile parameters ΔT_1 (^{0}C) in rectangular simply supported panels. *(a)* For specimen A (b) for specimen B, and (c) for specimen C.

The three regions shown in Fig 5 (a) are characterized by the values of the eigenvalue ($-\gamma^2$), as given in the following Table,

Region	$-\gamma^2$	Type of Motion
1	Real & positive	Steady oscillation, $\omega=\gamma_I$
2	Complex	Complex roots $\pm(\gamma_r+j\gamma_I)$, one root lead to divergent panel oscillations ($\gamma_r>0$)
3	Negative	Exponential divergence, $\gamma_r>0$ and $\gamma_I=0$

3.2 Effect for flow direction on flutter boundary

The variation of critical dynamic pressures with angle of flow θ (w.r.t edge *a)* is shown in Fig 6. It can be observed that for all thermal profiles in rectangular panels, the flow along the longer edge is most critical, *i.e.* the critical dynamic pressure is the lowest for flow along the longer edge of the panel.



(a) **(b)** **(c)**

Figure 6. Variation of critical dynamic pressure q_{cr} (in N/m^2), with flow angle θ (in degrees) for various thermal profile parameters ΔT_1 (^{0}C) in rectangular simply supported panels. *(a)* For square panel (specimen A), *(b)* For panel of aspect ratio *a/b*=2 (Specimen B), and (c) For panel of aspect ratio *a/b*=7.2 (Specimen C). Air density assumed for Specimens A and B are *1.225 kg/m^3*. Air density assumed for Specimen C is 0.715 *kg/m^3*.

4. CONCLUSIONS

The variation of critical dynamic pressure with flow direction and various parabolic thermal profiles of rectangular panels under supersonic flow is investigated for some given specimen panels, using in-house FEA code and NASTRAN. The results indicate that thermal profiles (having thermal gradients) induce in-plane compressive stresses in panels, even without any in-plane edge constraints, and effectively lower the flutter boundary. For a given thermal profile, flow along the longer side is most critical.

Results from the analytical formulation, in-house FEM code are in good agreement with those from NASTRAN software till the static thermal buckling point, beyond which NASTRAN results indicate a sharper fall of critical values with the thermal parameter, because NASTRAN considers the analysis as a post buckling geometrical nonlinear problem, whereas the analytical and in-house FEM code continues as a linear analysis.

ACKNOWLEDGMENTS

The authors are grateful to the Head, Mr D. V. Venkatasubramanyam, of Structural Technologies Division, and Dr A. R. Upadhaya, Director, NAL, for their generous support and encouragement for the work reported here.

REFERENCES

[1] Holt Ashley and Garabed Zartarian., Piston theory - a new aerodynamic tool for the aeroelastician, *J Aeronautical Sciences*, 1956, 23, 1109-1118
[2] Fung,Y.C.B., ***A summary of the theories and Experiments on panel flutter***, AFOSR TN, 1960, 60-224.
[3] Dowel E.H., Flutter of Multibay panels at high supersonic speeds, *J AIAA*, 1966, 4 (7), 1241-1248.
[4] Dowel E.H., ***Aeroelasticity of plates and shells*****,** Noordhoff, Lyden, the Netherlands.
[5] Ellen, C.H., Approximate Solutions of Membrane Flutter Problem, *J AIAA*, 1965, 3, 1186-1193.
[6] Harry G. Schaeffer and Walter L. Heard*,* Supersonic Flutter of a Thermally Stressed Flat Panel with Uniform Edge loads*, NASA TN*-D-3077, 1965.
[7] G.Sander, C.Bon and M.Geradin., Finite element analysis of Supersonic Panel Flutter, *J Numerical Methods in Engineering*, 1973, 7, 379-394.
[8] Zienkiewicz O. C, ***Finite Element Method for Solid and Structural Mechanics***, Elsevier, Butterworth Heinemann, 2000.
[9] S Mukherjee, Deepa N, Avinash R, M Manjuprasad and S Viswanath. Analytical investigation of supersonic panel flutter under thermal environment with arbitrary flow direction, *NAL Project Document ST 0704*, 2007.
[10] Somenath Mukherjee, M. Manjuprasad, R. Avinash, S. Deepa Sakravarthini. Finite element studies on supersonic panel flutter under high thermal environment with arbitrary flow direction, *TM-ST-07 01,* 2007.
[11] MSC.NASTRAN Combined Documentation 2005.